Gosford High School

2011
HIGHER SCHOOL
CERTIFICATE
TRIAL EXAMINATION

# Mathematics

## General Instructions

- Reading Time - 5 minutes.
- Working Time - 3 hours.
- Write using a blue or black pen.
- Board approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown for every question.
- Begin each question on a fresh sheet of paper.

Total marks (**120**)

- Attempt Questions 1-10.
- All questions are of equal value.

**Question 1** **(12 Marks)** Use a **SEPARATE** writing booklet

**Marks**

a) Evaluate $3e^{3.17}$ correct to 3 significant figures. 1

b) Solve $|2x-3| \geq 7$. 2

c) Expand and simplify $(2x-3y)^2 - 5x(x-2y)$. 2

d) If $f(x) = 2\sin 3x$ find the exact value of $f'\left(\frac{\pi}{18}\right)$ 2

e) Find the exact solutions of $2x^2 - x - 9 = 0$. 2

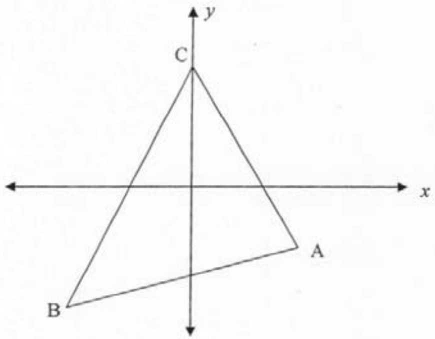
f) Factorise $6x^2 - 3xy - 4xz + 2yz$. 2

g) Evaluate $\log_5\left(\frac{1}{25}\right)$. 1

**Question 2** **(12 Marks)** Use a **SEPARATE** writing booklet

**Marks**

a)




$A(2,-2)$, $B(-2,-3)$ and $C(0,2)$ are the vertices of a triangle as shown.

(i) Find the length of AC. 1

(ii) Find the gradient of AC. 1

(iii) Find the equation of the line AC in general form. 2

(iv) Calculate the perpendicular distance from B to AC and hence the area of triangle ABC. 3

(v) Find the coordinates of D such that ABCD is a parallelogram.. 1

(vi) What angle does the line AC make with the positive direction of the $x$ axis (answer to the nearest minute). 1

b) Find

(i) $\int \frac{x^2}{x^3+1}\,dx$ 1

(ii) $\int_1^2 \frac{x^3+1}{x^2}\,dx$ 2

**Question 3** **(12 Marks)** Use a **SEPARATE** writing booklet

**Marks**

a) Find :

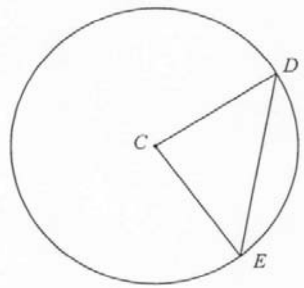
(i) $\frac{dy}{dx}$ when $y = \frac{1}{3-x}$. 1

(ii) $\frac{d}{dx}(2\tan 3x)$ 1

(iii) the derivative of $\frac{e^{2x}+4}{x^3}$. 2

(iv) $f'(x)$ if $f(x) = (x^3-2x).\ln(x)$. 2

b) A circle has centre $C$ and radius 12 cm. The length of the arc $DE$ is $2\pi$ cm.




NOT TO SCALE

(i) Find $\angle DCE$ (in radians). 1

(ii) Find the area of the minor segment cut off by the chord $DE$. 2

c) The gradient function of a certain curve is given by $\frac{dy}{dx} = \cos x - 2\sin 2x$. 3

If the curve passes through the origin find $y$ when $x = \frac{\pi}{2}$.

**Question 4** **(12 Marks)** Use a **SEPARATE** writing booklet

Marks

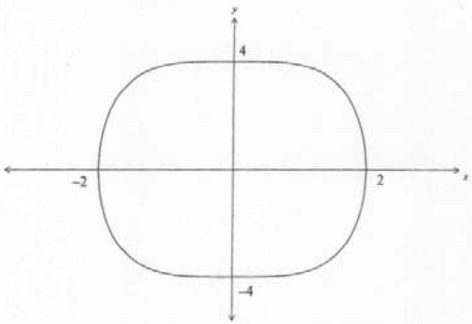
a) For the sequence 100, 95, 90, 85.....

(i) Show the sequence is Arithmetic. 1

(ii) Find the 25th term. 1

(iii) How many terms are needed to give a sum of zero? 2

b) The sketch shows the curve $y^2 + x^4 = 16$. 2




The area enclosed within the curve is rotated about the $x$ axis. Find the volume of the solid of revolution so formed.

c) Find the gradient of the normal to the curve $y = x\ln x$ at the point on the curve where $x = 1$. 2

d) Draw a sketch showing the graph of $y = 2\sin 3x$ for $0 \le x \le 2\pi$ 2

e) A die is rolled 3 times. What is the probability that:

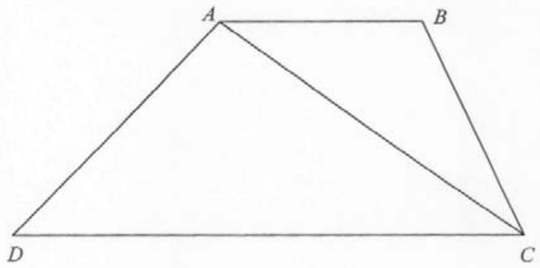
(i) No sixes are rolled? 1

(ii) At least 1 six is rolled? 1

**Question 5** **(12 Marks)** Use a **SEPARATE** writing booklet

Marks

a) The figure $ABCD$ is a trapezium. $\angle ABC = \angle DAC$. $AB = 12$ cm, $AD = 20$ cm and $AC = 24$ cm.




(i) Prove that $\Delta ABC \,|||\, \Delta ADC$. 2

(ii) Calculate the length of $BC$. 2

b) (i) For the function $f(x) = e^x \ln x$ copy and complete the following table of values. (give your answer correct to 1 decimal place). 2

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $f(x)$ | | | | | |

(ii) Use Simpson's rule with these 5 values to find an approximate value of $\int_1^5 e^x \ln x \, dx$. 2

c) A particle moves so that its displacement, $x$ metres, from the origin at time, $t$ seconds, is given by:

$$x = t^3 - 6t^2 + 9$$

(i) Find the initial displacement. 1

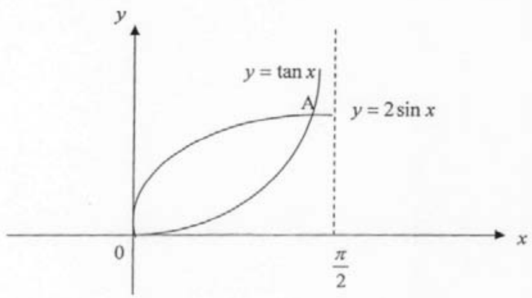
(ii) Find when the particle is at rest. 2

(iii) Find the distance travelled in the first 4 seconds. 1

**Question 6** **(12 Marks)** Use a **SEPARATE** writing booklet **Marks**

a) Draw a sketch graph of the function $y = x^4 - 8x^3$, for the domain $-1 \le x \le 9$, showing any intercepts, turning points and inflections and the end points for the domain. 6

b)




The diagram shows the curves $y = 2\sin x$ and $y = \tan x$ for $0 \le x \le \frac{\pi}{2}$. A is the point of intersection of the two curves.

(i) Show by substitution that the coordinates of A are $\left(\frac{\pi}{3}, \sqrt{3}\right)$ 1

(ii) Show that $\frac{d}{dx}(\ln(\cos x)) = -\tan x$ 1

(iii) Hence find the area between the two curves. 2

c) The area under the curve $y = \frac{1}{x}$ from $x = 1$ to $x = b$ is 1 square unit. What is the value of b? 2

**Question 7** **(12 Marks)** Use a **SEPARATE** writing booklet **Marks**

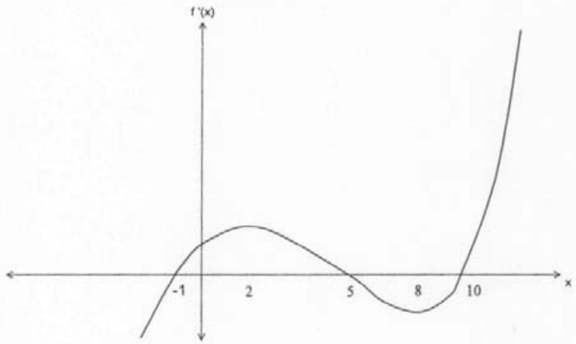
a) A plane which is initially flying at a speed of 200 m/s cuts its engines. It then decelerates at a rate which is proportional to its speed at that time. (i.e. $\frac{dS}{dt} = -kS$ ).

(i) Show that $S = S_0 e^{-kt}$ is a solution to this equation. 1

(ii) After 50 seconds its speed is 120 m/s. Find the values of $S_0$ and of $k$. 2

(iii) Find the speed after 90 seconds. 1

b) The graph of $y = f'(x)$ is shown below.




(i) Give the $x$ values for the turning points of $y = f(x)$. 1

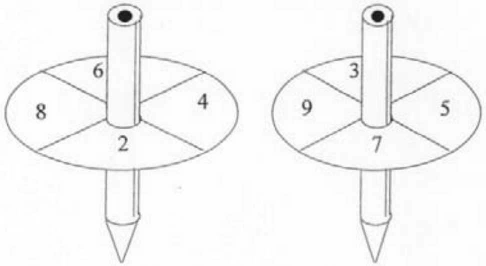
(ii) Draw a sketch of $y = f(x)$. 2

(iii) Draw a sketch of $y = f''(x)$. 2

**Question 7 continues.**

**Question 7 continued.** **Marks**

c) In a game, on a single turn a player whirls the two spinners shown below, each of which has 4 possible numbers. The score for the turn is obtained by subtracting the score on the second spinner from that on the first.




Spinner 1 Spinner 2

(i) List the possible outcomes for the scores on a single turn. 2

(ii) What is the probability of scoring a negative number on a single turn? 1

**Question 8** **(12 Marks)** Use a **SEPARATE** writing booklet **Marks**

a) (i) Differentiate $\cos^2 3x$ with respect to $x$. 2

(ii) Hence evaluate $\int_{\frac{\pi}{6}}^{\pi} \cos 3x \sin 3x\,dx$ . 3

b) Sue is raising money for charity by jumping on a Pogo Stick. Her challenge is to jump between two points, A and B, twenty times. On her first attempt she takes 45 jumps. On her second attempt she takes 48 jumps. On her third attempt she takes 51 jumps. She continues this pattern for all 20 attempts.

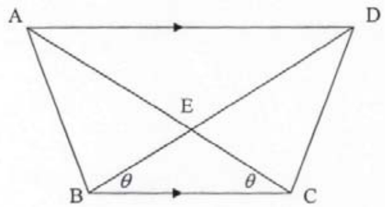
(i) How many jumps did she make on her $20^{th}$ attempt? 2

(ii) How many jumps did she make altogether? 2

c) The curve $y = ax^3 + \frac{b}{x^2}$ cuts the $x$- *axis* at $x = 1$ and the gradient of the tangent at this point is 2. Find the values of $a$ and $b$. 3

**Question 9** **(12 Marks)** Use a **SEPARATE** writing booklet **Marks**

a)

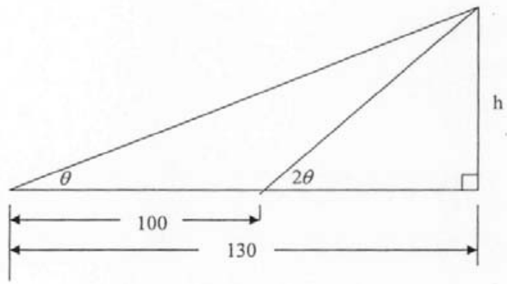



In the diagram AD is parallel to BC and angle DBC equals angle ACB.

(i) Show that AE = DE. 2

(ii) Prove that triangle ABC is congruent to triangle DCB. 3

b)




When at a distance of 130 metres from the foot of a tower its angle of elevation was observed; on walking 100 metres nearer the angle of elevation was doubled.

(i) Find the height, (h), of the tower correct to the nearest centimetre. 2

(ii) Find, correct to the nearest minute, the first angle of elevation, $\theta$ . 2

c) Prove that $\sin\theta + 1 + \cos\theta \cot\theta - \operatorname{cosec}\theta = 1$ 2

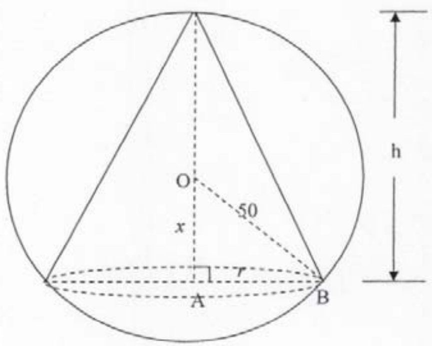
**Question 10 (12 Marks)** Use a **SEPARATE** writing booklet **Marks**

a) A spherical balloon which initially has a radius of 20 cm, is inflated so that its volume increases at a constant rate of $100\,cm^3\,/\,second$.

(i) Find an expression for the volume, $V$, at any time. **2**

(ii) Calculate the time required, to the nearest second, for the volume to reach 100 litres ( $1000\,cm^3 = 1\,litre$ ). **2**

b)




The diagram shows a cone of base radius $r\ cm$ and height $h\ cm$ inscribed in a sphere of radius 50 $cm$. The centre of the sphere is O and angle OAB $= 90^\circ$. Let OA $= x\ cm$.

(i) Show that $r = \sqrt{2500 - x^2}$ **2**

(ii) Hence show that the volume $V\ cm^3$, of the cone is given by:

$$V = \frac{\pi}{3}\left(2500 - x^2\right)\left(50 + x\right).$$

**2**

(iii) Find the radius of the largest cone which can be inscribed in the sphere. (Give your answer correct to the nearest $mm$.) **4**

**End of Examination**

Gosford High School

# Mathematics 2011 Trial Solutions.

Q1) a) $3e^{3\cdot 17}$

$= 71.4$

b) $|2x-3| \geq 7$

$2x-3 \leq -7$ or $7 \leq 2x-3$

$2x \leq -4$ or $10 \leq 2x$

$x \leq -2$ or $5 \leq x$

c) $(2x-3y)^2 - 5x(x-2y)$

$4x^2 - 12xy + 9y^2 - 5x^2 + 10xy$

$9y^2 - x^2 - 2xy$

d) $f(x) = 2\sin 3x$

$f'(x) = 6\cos 3x$

$f'(\frac{\pi}{18}) = 6\cos(\frac{\pi}{6})$

$= 6 \cdot \frac{\sqrt{3}}{2}$

$= 3\sqrt{3}$.

e) $2x^2 - x - 9 = 0$

$x = \frac{1 \pm \sqrt{1+72}}{4}$

$= \frac{1 \pm \sqrt{73}}{4}$

f) $6x^2 - 3xy - 4xz + 2yz$

$= 3x(2x-y) - 2z(2x-y)$

$= (2x-y)(3x-2z)$

g) $\log_5(\frac{1}{25})$

$= \log_5 5^{-2}$

$= -2\log_5 5$

$= -2$

Q2) a) $A(2,-2)$, $B(-2,-3)$, $C(0,2)$

i) $AC = \sqrt{(2-0)^2 + (-2-2)^2}$

$= \sqrt{4+16}$

$= 2\sqrt{5}$

ii) $m_{AC} = \frac{2+2}{0-2}$

$= -2$

iii) $y - 2 = -2(x-0)$

$y - 2 = -2x$

$2x + y - 2 = 0$

iv) $D = \left|\frac{2\times -2 + 1\times -3 - 2}{\sqrt{2^2+1^2}}\right|$

$= \left|\frac{-9}{\sqrt{5}}\right|$

$= \frac{9}{\sqrt{5}}$  $(\frac{9\sqrt{5}}{5})$

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times \frac{9}{\sqrt{5}} \times 2\sqrt{5}$

$= 9$ sq units

v) $D(4, 3)$

vi) $\tan\theta = m$

$\tan\theta = -2$

$\theta = 116°34'$

b) i) $\int \frac{x^2}{x^3+1}\,dx$

$= \frac{1}{3}\ln(x^3+1) + c$

ii) $\int_1^2 \frac{x^3+1}{x^2}\,dx$

$= \int_1^2 \frac{x^3}{x^2} + \frac{1}{x^2}\,dx$

$= \int_1^2 x + x^{-2}\,dx$

$= \left[\frac{x^2}{2} - \frac{1}{x}\right]_1^2$

$= (2 - \frac{1}{2}) - (\frac{1}{2} - 1)$

$= 2.$

Q3) a) i) $\frac{d}{dx}\left(\frac{1}{3-x}\right)$

$= \frac{1}{(3-x)^2}$

ii) $\frac{d}{dx}(2\tan 3x)$

$= 6\sec^2 3x$

iii) $\frac{d}{dx}\left(\frac{e^{2x}+4}{x^3}\right)$

$= \frac{2x^3e^{2x} - 3x^2(e^{2x}+4)}{x^6}$

$= \frac{2x^3e^{2x} - 3x^2e^{2x} - 12x^2}{x^6}$

$= \frac{2xe^{2x} - 3e^{2x} - 12}{x^4}$

iv) $f(x) = (x^3-2x)\ln x$

$f'(x) = \frac{x^3-2x}{x} + (3x^2-2)\ln x$

$= x^2 - 2 + (3x^2-2)\ln x$



b) i) $l = r\theta$

$2\pi = 12\theta$

$\theta = \frac{\pi}{6}$

ii) $A = \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$

$= \frac{1}{2}\times 12^2 \times \frac{\pi}{6} - \frac{1}{2}\times 12^2 \times \frac{1}{2}$

$= 12\pi - 36$ cm$^2$

$(\doteqdot 1.7$ cm$^2)$

c) $\frac{dy}{dx} = \cos x - 2\sin 2x$

$y = \sin x + \cos 2x + c$

$0, y=0 \therefore 0 = 0 + 1 + c$

$\therefore y = \sin x + \cos 2x - 1$

$x = \frac{\pi}{2}$: $y = 1 - 1 - 1$

$y = -1$

Q4) a) 100, 95, 90, ...

i) A.P. if

$95 - 100 = 90 - 95$

$-5 = -5$

True $\therefore$ A.P.

ii) $T_{25} = 100 + (25-1)\times -5$

$= 100 - 120$

$= -20$

iii) $S_n = \frac{n}{2}(2a + (n-1)d)$

$0 = \frac{n}{2}(200 + (n-1)-5)$

$0 = \frac{n}{2}(200 - 5n + 5)$

$0 = n(205 - 5n)$

$n = 0, 41$

$n = 0$ not a solution

$\therefore$ 41 terms

needed…

b) $V = \pi\int_{-2}^{2} y^2\,dx$

$= 2\pi\int_0^2 16 - x^4\,dx$

$= 2\pi\left[16x - \frac{x^5}{5}\right]_0^2$

$= 2\pi\left((32 - \frac{32}{5}) - 0\right)$

$= \frac{256\pi}{5}$ cubic units.

c) $y = x\ln x$

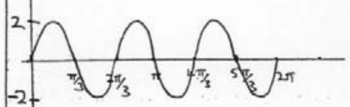
$\frac{dy}{dx} = x \times \frac{1}{x} + \ln x$

$= 1 + \ln x$

at $x = 1$: $\frac{dy}{dx} = 1 + 0$

$\therefore$ gradient normal $= -1$

d) $y = 2\sin 3x$.




e) i) P(no sixes)

$= \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6}$

$= \frac{125}{216}$

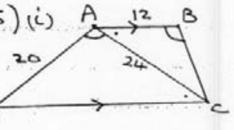
ii) P(at least 1 six)

$= 1 - P$(no sixes)

$= 1 - \frac{125}{216}$

$= \frac{91}{216}$.

Q5) (i)




ABCD a trapezium

$\Rightarrow AB \parallel DC$.

$\Delta ABC \,|||\, \Delta ADC$

$\angle ABC = \angle DAC$ given

$\angle BAC = \angle ACD$

alternate angles in parallel lines

$\therefore \Delta ABC \,|||\, \Delta ADC$ (A.A.A)

ii) $\frac{BC}{20} = \frac{12}{24}$

$BC = 10$

b) i) $f(x) = e^x \ln x$

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $f(x)$ | 0 | 5.1 | 22.1 | 75.7 | 238.9 |

ii) $A = \frac{1}{3}[1^{st} + last + 2\times even + 4\times odd]$

$= \frac{1}{3}\{0 + 238.9 + 2\times 22.1 + 4(5.1 + 75.7)\}$

$= \frac{1}{3}\{238.9 + 44.2 + 323.2\}$

$= \frac{1}{3} \times 606.3$

$= 202.1$

c) $x = t^3 - 6t^2 + 9$

i) initial when $t = 0$

$x = 9$ m.

ii) at rest $v = 0$.

$v = \frac{dx}{dt} = 3t^2 - 12t$

$\therefore 3t^2 - 12t = 0$

$3t(t-4) = 0$

$t = 0, 4$

iii) $t = 4$

$x = 4^3 - 6\times4^2 + 9$

$= 64 - 96 + 9$

$= -23$

$\therefore$ distance travelled

$= 9 + 23$

$= 32$ m.

Q6)

a) $y = x^4 - 8x^3$.

$y' = 4x^3 - 24x^2$

$y'' = 12x^2 - 48x$

S.P. $y' = 0$

$4x^3 - 24x^2 = 0$

$4x^2(x-6) = 0$

$x = 0, \quad x = 6$

$y = 0, \quad y = -432$

$f''(0) = 0$

$\therefore (0,0)$ possible horizontal point of inflexion

$f''(6) > 0$

$\therefore (6,-432)$ min

t. of inflexion $f''(x) = 0$.

$12x^2 - 48x = 0$

$12x(x-4) = 0$

$x = 0, \quad x = 4$

| $x$ | -1 | 0 | 1 |
|---|---|---|---|
| $y''$ | + | 0 | - |

| $x$ | 3 | 4 | 5 |
|---|---|---|---|
| $y''$ | - | 0 | + |

change in concavity

$\therefore (0,0), (4,-256)$ are points of inflexion

end points in the given domain

$(-1,9) \quad (9,729)$

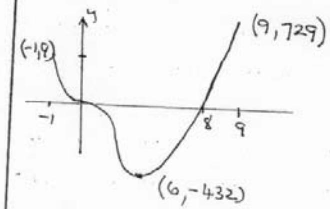
'x' intercepts. $(y=0)$

$x^4 - 8x^3 = 0$

$x^3(x-8) = 0$

$\therefore$ x intercepts, 0, 8.

'y' intercepts, 0.




b) i) $y = \tan x$, $y = 2\sin x$ at A $(\frac{\pi}{3}, \sqrt{3})$

$y = \tan x$

$\sqrt{3} = \tan\frac{\pi}{3}$

$\sqrt{3} = \sqrt{3}$

true.

$y = 2\sin x$

$\sqrt{3} = 2\sin\frac{\pi}{3}$

$\sqrt{3} = 2\times\frac{\sqrt{3}}{2}$

$\sqrt{3} = \sqrt{3}$

True

as A lies on both curves it must be a point of intersection

ii) $\frac{d}{dx}(\ln(\cos x)) = -\tan x$

LHS $= \frac{-\sin x}{\cos x}$

$= -\tan x$

$=$ RHS.

iii) $A = \int_0^{\pi/3} 2\sin x - \tan x \, dx$

$= \left[-2\cos x + \ln(\cos x)\right]_0^{\pi/3}$

$= (-2\times\frac{1}{2} + \ln\frac{1}{2}) - (-2 + \ln 1)$

$= -1 + \ln\frac{1}{2} + 2$

$= 1 + \ln 2^{-1}$

$= 1 - \ln 2$.

c) $\int_1^b \frac{1}{x}\,dx = 1$

$\left[\ln x\right]_1^b = 1$

$\ln b - \ln 1 = 1$

$\log_e b = 1$

$b = e^1$

$\therefore b = e$.

Q7) i) $S = S_0 e^{-kt}$

$\frac{dS}{dt} = -kS_0 e^{-kt}$

$= -kS$

ii) $t = 0, \ S = 200$

$200 = S_0 e^0$

$S_0 = 200$



$S = 200 e^{-kt}$

$t = 50 \quad S = 120$

$120 = 200 e^{-50k}$

$0.6 = e^{-50k}$

$\ln(0.6) = -50k$

$k = \frac{\ln(0.6)}{50}$

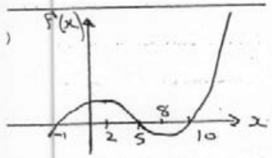
$k \doteqdot 0.01022$.

ii) $S = 200 e^{-0.0102t}$

$t = 90$

$S = 200 e^{-0.91949}$

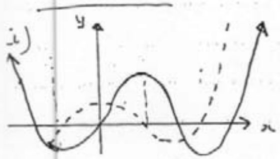
$S = 79.7$ m/s.



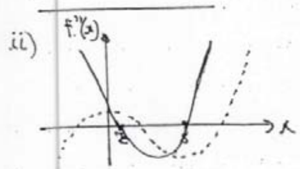

i) turning pts. when $f'(x) = 0$.

ie $x = -1, 5, 10$.

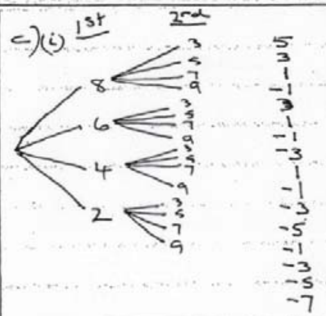
i)




ii)




c)(i)




ii) $\frac{10}{16}$

$= \frac{5}{8}$

Q8) a)(i) $\frac{d}{dx}(\cos^2 3x)$

$= 2\cos 3x \cdot -3\sin 3x$

$= -6\sin 3x\cos 3x$.

ii) $\int_{\pi/6}^{\pi} \cos 3x \sin 3x \, dx$

$= -\frac{1}{6}\int_{\pi/6}^{\pi} -6\cos 3x \sin 3x \, dx$

$= -\frac{1}{6}\left[\cos^2 3x\right]_{\pi/6}^{\pi}$

$= -\frac{1}{6}\left(\cos^2(3\pi) - \cos^2(\frac{\pi}{2})\right)$

$= -\frac{1}{6}\left((-1)^2 - 0\right)$

$= -\frac{1}{6}$

b) 45, 48, 51 ....

i) $T_{20} = 45 + 19\times3$

$= 45 + 57$

$= 102$

ii) $S_n = \frac{n}{2}(a + L)$

$= \frac{20}{2}(45 + 102)$

$= 10\times147$

$= 1470$

c) $y = ax^3 + \frac{b}{x^2}$

$\frac{dy}{dx} = 3ax^2 - \frac{2b}{x^3}$

cuts 'x' axis at $x = 1$

ie $x = 1, \ y = 0$

$0 = a + b \quad$ ---(1)

at $x = 1 \quad \frac{dy}{dx} = 2$

$2 = 3a - 2b \quad$ --(2)

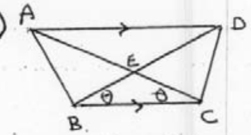
(1)×2. $0 = 2a + 2b \quad$ --(3)

(2)+(3) $2 = 5a$

$\frac{2}{5} = a$

Sub into (1) $b = -\frac{2}{5}$

Q9)




$\angle EBC = \angle ECD$ (given)

i) $\angle EDA = \angle EBC$

and $\angle DAE = \angle ECB$

(alternate angles in parallel lines)

$\therefore \angle EDA = \angle DAE$

$\therefore \Delta EAD$ is isosceles - base angles equal

$\therefore AE = DE$.

ii) $\Delta EBC$ is isosceles

given $\angle EBC = \angle ECB$.

$\therefore EB = EC$

from (i) $AE = DE$

$\therefore AC = DB$

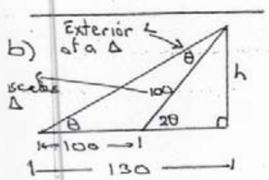
$\angle DBC = \angle ACB$

(given)

BC is common

$\therefore \Delta ABC \equiv \Delta DCB$ (SAS)

b)




i) $h^2 = 100^2 - 30^2$

$= 9100$

$h = 95.39\,m.$

i) $\tan\theta = \dfrac{h}{130}$

$= \dfrac{95.39}{130}$

$= 36^\circ 16'$

:)

$n\theta + 1 + \cos\theta . \cot\theta - \operatorname{cosec}\theta = 1$

L.HS

$n\theta + 1 + \cos\theta . \dfrac{\cos\theta}{\sin\theta} - \dfrac{1}{\sin\theta}$

$\dfrac{\ln^2\theta + \sin\theta + \cos^2\theta - 1}{\sin\theta}$

$1 + \sin\theta - 1$

$= \dfrac{\sin\theta}{\sin\theta}$

$= 1$

$= RHS$

(Q10)

i) $\dfrac{dv}{dt} = 100$

$v = 100t + c$

at $t = 0$ $V = \frac{4}{3} \times \pi \times 20^3$

$= 33510.3\,cm^3$

$\therefore c = 33510.3$

$\therefore V = 100t + 33510$

ii) $V = 100\,000$

$V = 100t + 33510$

$100\,000 = 100t + 33510$

$100t = 66490$

$t = 664.9$ sec

$= 665$ sec.

b) i) by pythagoras

$r^2 = 50^2 - x^2$

$r = \sqrt{2500 - x^2}$

ii) $V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi (2500 - x^2)(50 + x)$

$V = \frac{\pi}{3}(2500 - x^2)(50 + x)$

$\dfrac{dV}{dx} = \frac{\pi}{3}\left((2500 - x^2) + (50 + x)x - 2x\right)$

$= \frac{\pi}{3}(2500 - x^2 - 100x - 2x^2)$

$= \frac{\pi}{3}(2500 - 100x - 3x^2)$

$\dfrac{d^2V}{dx^2} = \frac{\pi}{3}(-100 - 6x)$

max when $\dfrac{dV}{dx} = 0$.

$\frac{\pi}{3}(2500 - 100x - 3x^2) = 0$

$3x^2 + 100x - 2500 = 0$

$x = \dfrac{-100 \pm \sqrt{40000}}{6}$

$= \dfrac{-100 \pm 200}{6}$

$= \dfrac{100}{6}$ or $-\dfrac{300}{6}$ ↘ not a solution

$f''\left(\frac{100}{6}\right) < 0$

$\therefore$ max $x = \dfrac{100}{6}$

Now $r = \sqrt{2500 - x^2}$

$= \sqrt{2500 - (16.7)^2}$

$r = 46.8\,cm.$